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INVESTIGATION OF HEAT TRANSFER BEHIND AN OSCILLATING GRID

W. KÜHN
Institut für Mechanik der Akademie der Wissenschaften der DDR, Rudower Chaussee 5, 1199 Berlin, D.D.R.

Abstract—Measurements of heat transfer behind a harmonically oscillating grid yielded rates of heat spread which were higher than those in usual turbulent flows produced by stationary grids. The mean temperature profiles showed an unexpected shape with two maxima and a minimum between them. This was caused by large-scale velocity fluctuations which produce heat fluxes independent of the local mean temperature gradient. A simple heat transfer model is proposed which takes into account both the large-scale velocity fluctuations produced by the oscillating grid and the 'usual' turbulence. The calculated mean temperature profiles show a good agreement with the measured ones.

NOMENCLATURE

$\tilde{A}, \tilde{h}; A, h$	parameters of the particle distribution in the system $\tilde{x}_1, \tilde{x}_2$ and x_1, x_2, respectively
f	frequency
p	probability density
r	amplitude of grid oscillations
S	Strouhal number, $\hat{v}_G/\bar{u}_\infty$
T	period of the grid oscillations
t	time
t_D	diffusion time of particles
$\bar{u}_\infty$	mean velocity of the outer flow which is not disturbed by the heat source
u, v	components of the velocity in the x and y directions
v'_s	non-stochastic velocity fluctuations in the y-direction which are produced by the grid oscillations
v_G	velocity of the oscillating grid
x_1, x_2	Cartesian system of coordinates (Fig. 5)
$\tilde{x}_1, \tilde{x}_2$	moving system of coordinates (Fig. 5)
x, y, z	Cartesian system of coordinates used in the test section (Fig. 1)
Y	distance in y-direction which is traversed by a particle in time t

Greek symbols

α	angle (Fig. 5)
α_G	angle of attack of the oscillating grid vanes
θ	temperature difference to the non-heated flow
θ_m	maximum value of a $\bar{\theta}$ profile along the y axis
θ_0	(constant) temperature of heated particles
σ	standard deviation of a mean temperature profile, $\int_{-\infty}^{+\infty} \bar{\theta}(y) y^2 \, dy / \int_{-\infty}^{+\infty} \bar{\theta}(y) \, dy$
τ	portion of time T during which particles with $\theta = \theta_0$ pass a point
τ_s	sum of the values $\tau(x_1, x_2)$ for the several wires of the heat source used
ω	$2\pi f$

Other symbols

$\bar{(\ )}$	time-mean value of ( )
$\hat{(\ )}$	maximum value of ( )
$(\ )'$	fluctuation of ( ) (deviation from the mean value)
$\langle(\ )\rangle$	averaged value of ( ) for many particles

1. INTRODUCTION

AN IMPORTANT task in the field of heat and mass transfer is to manage a rapid spread of heat or mass. The capability of a turbulent flow to transfer heat and the coefficient of turbulent heat transfer (eddy conductivity) depend on the macrostructure and the intensity of turbulence. The eddy conductivity increases with enlarging macroscales and as the intensity of the turbulent velocity fluctuations increase. In simple flows it is possible to determine the relation between the eddy conductivity or the mixing lengths for turbulent heat transfer on the one hand, and special integral length scales and the intensity of the components of the velocity fluctuations on the other [1].

Stationary grids produce turbulence with low heat transfer capabilities. The first reason for this is that the macroscales of the generated turbulence are of the order of the characteristic dimensions of the grid used, but these dimensions are restricted by the given flow field. The second disadvantage of common stationary grids is the fast decay of velocity fluctuations behind them. Therefore, investigations of the heat transfer properties of oscillating grids have been performed. These grids produce velocity fluctuations with comparatively large macroscales. Thus the decay of the turbulence is rather small. The generated turbulence is highly anisotropic because the velocity component parallel to the direction of the grid oscillations contains the main part of the turbulent energy. Following an

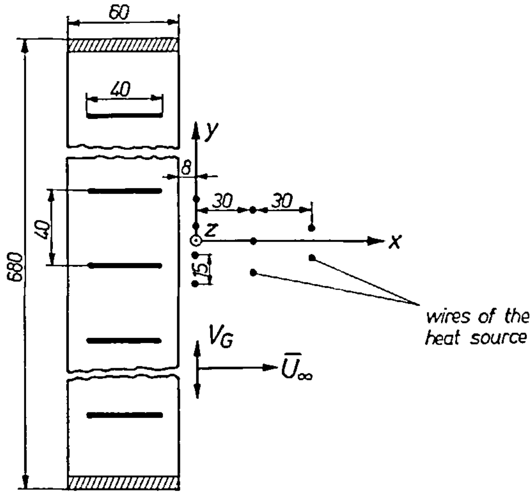

FIG. 1. The used oscillating grid and the heat source (dimensions in mm).

idea by M. Hoffmeister, the first investigations of the behaviour of the flow field behind oscillating grids were performed by Klatt [2].

2. EXPERIMENTAL APPARATUS

The measurements were performed in an open-jet wind tunnel. The nozzle exit had a 0.58 × 0.58 m^2 square cross-section. The oscillating grid used was situated at the exit of the nozzle. The grid had plane vanes of 1 mm thick wood which were parallel to the z-axis (Fig. 1). The grid oscillated harmonically in the y-direction, i.e. perpendicularly to the mean flow.

The heat source consisted of 9 electrically heated wires with a diameter of about 1.5 mm. The wires were oriented along the z-axis.

The measurements were carried out at a distance from the heat source, where the wake was sufficiently small to neglect it. At worst, the depth of the wake amounted to approximately 15% of the undisturbed outer flow velocity. The temperature differences were small in the flow field, thus no buoyancy effects could be detected. Mean temperatures and mean velocities exhibited plane distributions along the z-axis. The temperature was measured by thermocouples, the mean velocity by a Pitot tube.

3. EXPERIMENTAL RESULTS

The measurements were performed at different frequencies and grid oscillation amplitudes. The mean flow velocity was also changed. Thus the main flow parameter, the maximum angle of attack of the moving vanes, $\hat{\alpha}_G$, had different values. According to Fig. 1, the following simple expression can be obtained:

$$\tan \hat{\alpha}_G = \frac{\hat{v}_G}{\bar{u}_\infty} = \frac{2\pi f r}{\bar{u}_\infty} \tag{1}$$

where $\hat{v}_G$ is the maximum grid velocity. The RHS of equation (1) in principle represents a Strouhal number because it contains the frequency and the amplitude of the grid oscillations as well as the mean velocity. Therefore $\tan \hat{\alpha}_G$ can be interpreted as the Strouhal number S of the oscillating flow.

In Figs. 2–4 some of the measured mean temperature distributions are plotted. In these diagrams the mean temperature is divided by its maximum value at several cross-sections, and the y-coordinate by the standard deviation of the corresponding mean temperature profile. The mean temperature distributions at low Strouhal numbers (Fig. 2) are similar to the expected Gaussian error curve. They are not very different from the self-preserving distribution behind a stationary

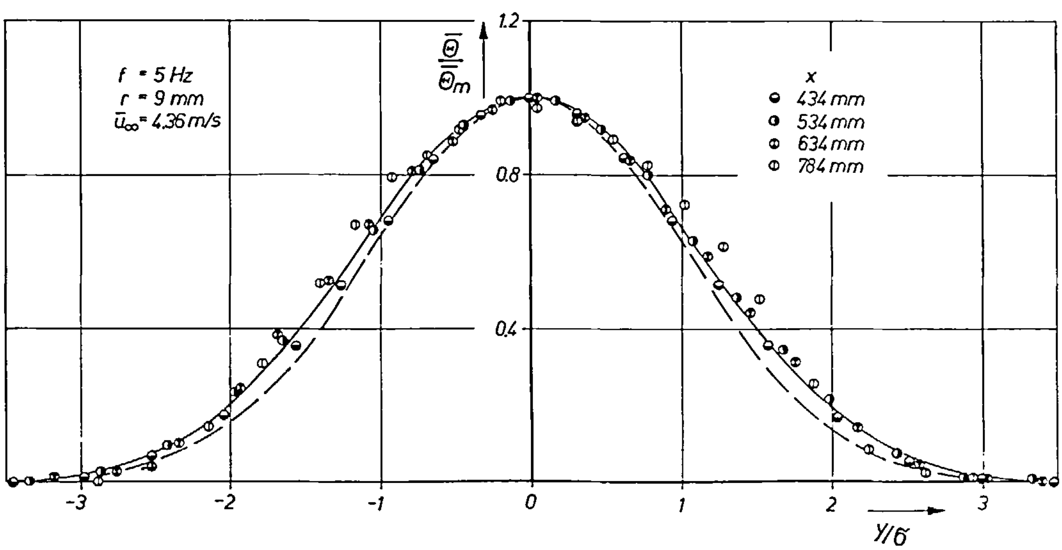

FIG. 2. Mean temperature distribution, $S = 0.0648$. (---- the self-preserving distribution behind a stationary parallel cylinder grid [5].)

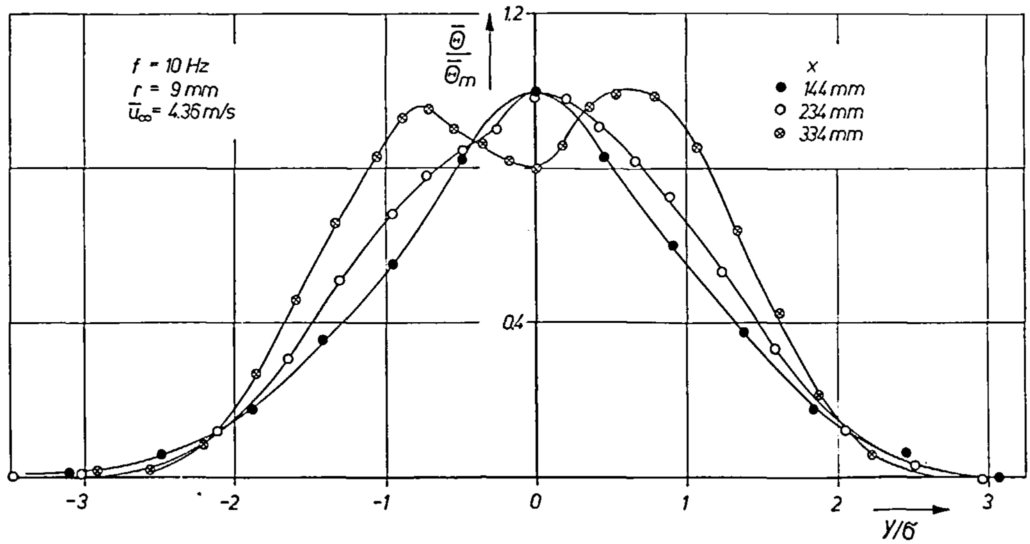

FIG. 3. Mean temperature distribution at small x, $S = 0.126$.

parallel cylinder grid [5] (centre-to-centre spacing of rods 30 mm, cylinder diameter 12 mm) which was also determined behind the described heat source. (See the dashed line in Fig. 2.) It can be seen, however, that the curves for different cross-sections are not strictly self-preserving. $\bar{\theta}/\bar{\theta}_m$ tends to increase with enlarging x in the region with $|y/\sigma| \approx 1$.

In Figs. 3 and 4, the normalized mean temperature distributions at a moderate Strouhal number are shown. These profiles mainly change their shape with increasing values of x. At short distances x, the temperature distribution is almost a Gaussian error curve, whereas farther downstream it displays two maxima with a minimum between them. The graph in Fig. 4 indicates that the wavy temperature distribution seems to be stable with increasing x.

4. A SIMPLE HEAT TRANSFER MODEL

The particle motion behind an oscillating grid shows two ranges: one with relatively small mixing lengths which corresponds to the 'usual' turbulent motion, and a second one with large mixing lengths which is produced by the oscillations of the grid. The usual turbulence in the first place originates from the production in the wake of the grid vanes. The influence of the molecular particle motion on the spread of heat can be neglected in the flows investigated.

The changing shape of the temperature profiles in Fig. 3 indicates that a strong heat transport against the mean temperature gradient in y-direction exists at moderate Strouhal numbers. This is caused by the large scale velocity fluctuations produced by the oscillations

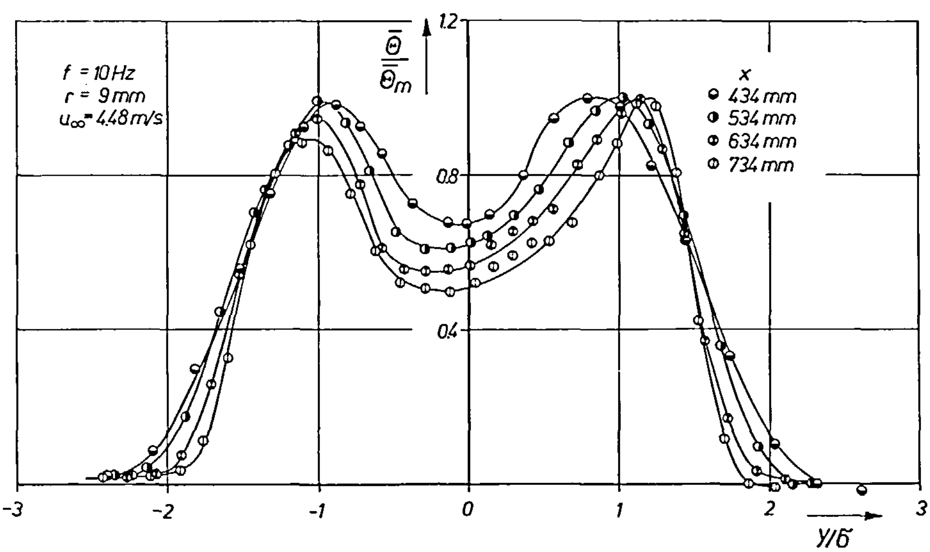

FIG. 4. Mean temperature distribution, $S = 0.126$.

of the grid. At small S, however, the influence of these large-scale movements of fluid lumps decreases and so the shape of the temperature distributions does not change remarkably along the test section used (Fig. 2). But the measured higher order moments (for instance, of temperature fluctuations) demonstrate the existence of a considerable bulk convection. (A report of these results is being prepared.) The intensity of the temperature fluctuations has maximum values about 100% higher than those behind the parallel cylinder grid mentioned above. It must be concluded that a model describing the heat transfer behind an oscillating grid has to take into account both the relatively small-scale turbulent motion and the bulk convection due to the large-scale velocity fluctuations.

Figure 5 shows a schematic diagram of an oscillating flow field behind a heated wire. The coordinate system $\tilde{x}_1, \tilde{x}_2$, whose origin coincides with the heated wire, is moving according to the instantaneous velocity fluctuations v_s' at the wire which are produced by the harmonic oscillations of the grid vanes. According to the plane oscillating grid it is possible to assume that the angle α does not depend on the y- and z-coordinates. The flow domains with the same α are similar to layers with large dimensions in the y- and z-direction and small ones in the x-direction. The velocity v_s' of these layers does not change remarkably with increasing x because they have a high inertia and, more importantly, they seem to be relatively stable. This can be concluded from flow visualizations by means of smoke [2] and is supported by the measured rate of heat spread which indicates the existence of large fluid elements preserving a nearly constant velocity over relatively long time periods (see Section 5). The shape of the filament lines corresponds to a sinusoid whose amplitude linearly increases with the x-coordinate (Fig. 6). Thus all particles emerging from the line heat source approximately move along straight lines, i.e. a particle passing the heat source under an angle α has the coordinate $\tilde{x}_2 = 0$ in a system $\tilde{x}_1, \tilde{x}_2$ which is inclined to $\bar{u}_\infty$ under the same angle. The additional turbulent motion, however, causes deviations from this straight motion along the $\tilde{x}_1$-axis. The problem arises to determine the distribution of the heated particles in the $\tilde{x}_1, \tilde{x}_2$ system which is obtained for many realizations of a certain angle α. In accordance with the behaviour of particles in turbulent flows which

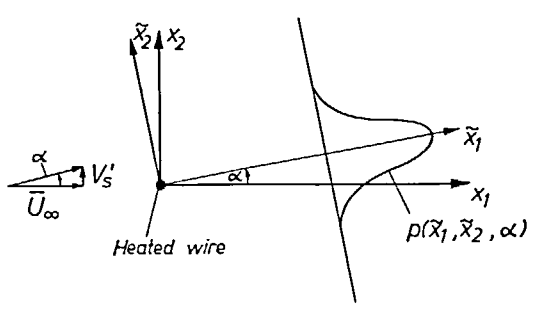

FIG. 5. Systems of coordinates in an oscillating flow field.

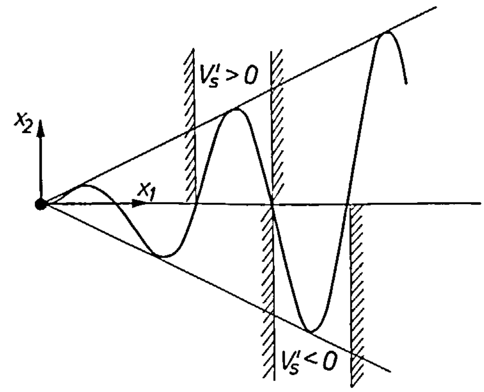

FIG. 6. A filament line in the investigated oscillating flows.

are generated by stationary grids, the normal error curve was assumed to be a probability density function [3],

$$p(\tilde{x}_1, \tilde{x}_2, \alpha) = \tilde{A}(\tilde{x}_1, \alpha)\,e^{-[\tilde{x}_2/\tilde{h}(\tilde{x}_1,\alpha)]^2 \ln 2}. \qquad (2)$$

$\tilde{A}$ or $\tilde{h}$ must be chosen according to the influence of the turbulent motion. The dependence on the angle α also becomes necessary if the turbulent disturbances are a function of α.

This can be caused, for instance, by a flow separation at the grid depending on the angle of incidence of the vanes.

The particles emerging from the heat source under a certain angle α are situated with the probability $p(\tilde{x}_1, \tilde{x}_2, \alpha)\,d\tilde{x}_1\,d\tilde{x}_2$ in an infinitesimal rectangle $d\tilde{x}_1\,d\tilde{x}_2$ which is located around a considered point Q with the coordinates $\tilde{x}_1, \tilde{x}_2$. This probability can be obtained if we consider many realizations of α,

$$p(\tilde{x}_1, \tilde{x}_2, \alpha)\,d\tilde{x}_1\,d\tilde{x}_2 = \frac{d\tau(\tilde{x}_1, \tilde{x}_2, \alpha)}{dt(\alpha)}. \qquad (3)$$

$d\tau(\tilde{x}_1, \tilde{x}_2, \alpha)$ represents the time interval during which the particles marked by the heat source are contained in the element $d\tilde{x}_1\,d\tilde{x}_2$ around the point $Q(\tilde{x}_1, \tilde{x}_2)$. $dt(\alpha)$ corresponds to the infinitesimal time interval during which the flow direction is located in the angle interval $d\alpha$ around α. The time interval $dt(\alpha)$ can be calculated by the spacetime motion of the grid. According to the harmonically oscillating grid we obtain

$$\frac{|d\alpha|}{dt(\alpha)} = 2\pi f \frac{\left(\dfrac{\hat{v}_s'^2}{\bar{u}_\infty^2} - \tan^2\alpha\right)^{1/2}}{1 + \tan^2\alpha}. \qquad (4)$$

The coordinates of the $\tilde{x}_1, \tilde{x}_2$-system can be expressed by those of the fixed system x_1, x_2 by the aid of the simple transformations

$$\begin{aligned} \tilde{x}_1 &= x_1 \cos\alpha + x_2 \sin\alpha, \\ \tilde{x}_2 &= -x_1 \sin\alpha + x_2 \cos\alpha. \end{aligned} \qquad (5)$$

Combining equations (2), (3) and (5), equation (6) for the time interval $d\tau$ can be obtained,

$$d\tau(x_1, x_2, \alpha) = dt(\alpha)A(x_1, x_2, \alpha)$$
$$\times\ e^{-[(-x_1 \sin\alpha + x_2 \cos\alpha)/h(x_1, x_2, \alpha)]^2 \ln 2}\ dx_1\ dx_2. \quad (6)$$

The addition of all intervals $d\tau$ during one period of the grid oscillation (T) yields the time portion of T during which heated particles pass the considered point Q (x_1, x_2). If it is assumed that all particles emerging from the heat source have approximately the same initial temperature difference θ_0 to the non-heated flow and preserve this temperature during the first time of flight the mean temperature is

$$\bar{\theta}(x_1, x_2) = \theta_0 \frac{\tau(x_1, x_2)}{T}. \quad (7)$$

The heat source used consisted of nine wires (Fig. 1). It was assumed that the heat supply at each wire is not influenced by the remaining wires. Thus the resulting time during which particles with $\theta = \theta_0$ are contained in the infinitesimal rectangle $dx\ dy$ around the point (x, y) is equal to the sum of the $\tau(x_1, x_2)$s of the different wires. Of course, the coordinates of the x, y-system have to be transformed into those of the x_1, x_2-systems of the various heated wires. Finally, the mean temperature may be written as

$$\bar{\theta}(x, y) = \frac{\tau_s(x, y)}{T}. \quad (8)$$

In Figs. 7(a)–(c), some results of the calculated mean temperature profiles are plotted. The calculations were performed numerically. They were based on the assumption that $\bar{A}(\tilde{x}_1, \alpha)$ and, consequently, $\bar{h}(\tilde{x}_1, \alpha)$ are constant. Of course, this is a rough approximation of the thermal spread in turbulent flows, but the comparison in Fig. 7 shows that the simple heat transfer model provides good agreement with the measured temperature profiles even if only estimations of the parameter h are available. ($\bar{\theta}_m$ can be determined from the power supply.) This feature of the model seems to be important for engineering purposes. More accurate calculations are possible if the spread of heat in the oscillating system is taken into account. Exact data concerning $\bar{h}$ and $\bar{A}$ become necessary for oscillating flows with high turbulent velocity fluctuations where the transfer due to the oscillations is small compared to that one performed by the 'usual' turbulence.

5. SOME SPECIAL HEAT TRANSFER FEATURES OF OSCILLATING FLOWS

In the following, some results derived from the measured mean temperature profiles will be discussed. In Fig. 8, the standard deviation of the temperature distributions is plotted as a function of the x-coordinate. The symbols used in Fig. 8 are explained in Table 1. The spread of heat continuously increases with increasing Strouhal number. The open triangles ($S = 0$) indicate that the molecular and turbulent heat diffusion are small if the oscillating grid is at rest. Of course, due to the shape of the heat source used, a certain initial amount of σ exists at small x.

In Fig. 8, the crosses show the standard deviation behind the parallel cylinder grid. This curve has an exponent of about 0.5 in accordance to Taylor's theory of turbulent diffusion [4]. On the other hand, the standard deviation of the oscillating flows is

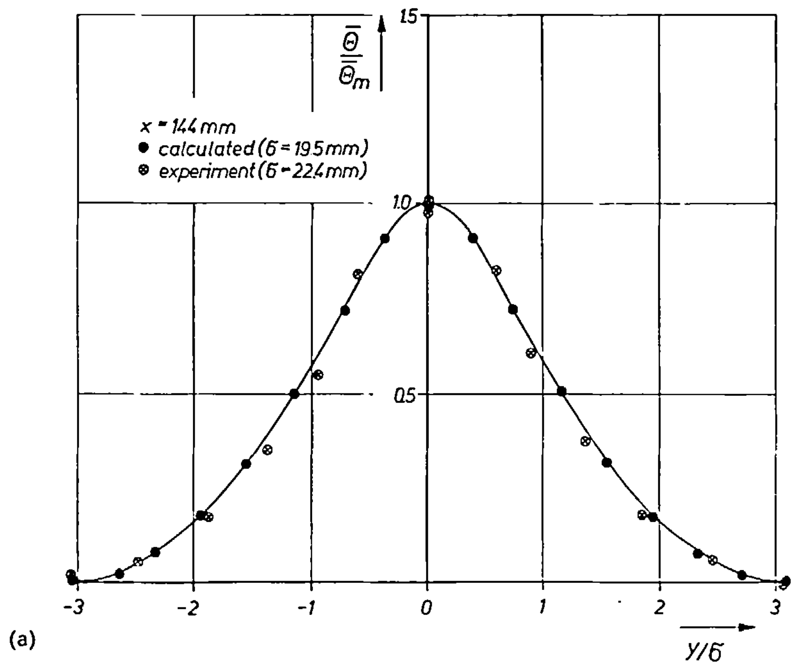

FIG. 7 (*continued overleaf*).

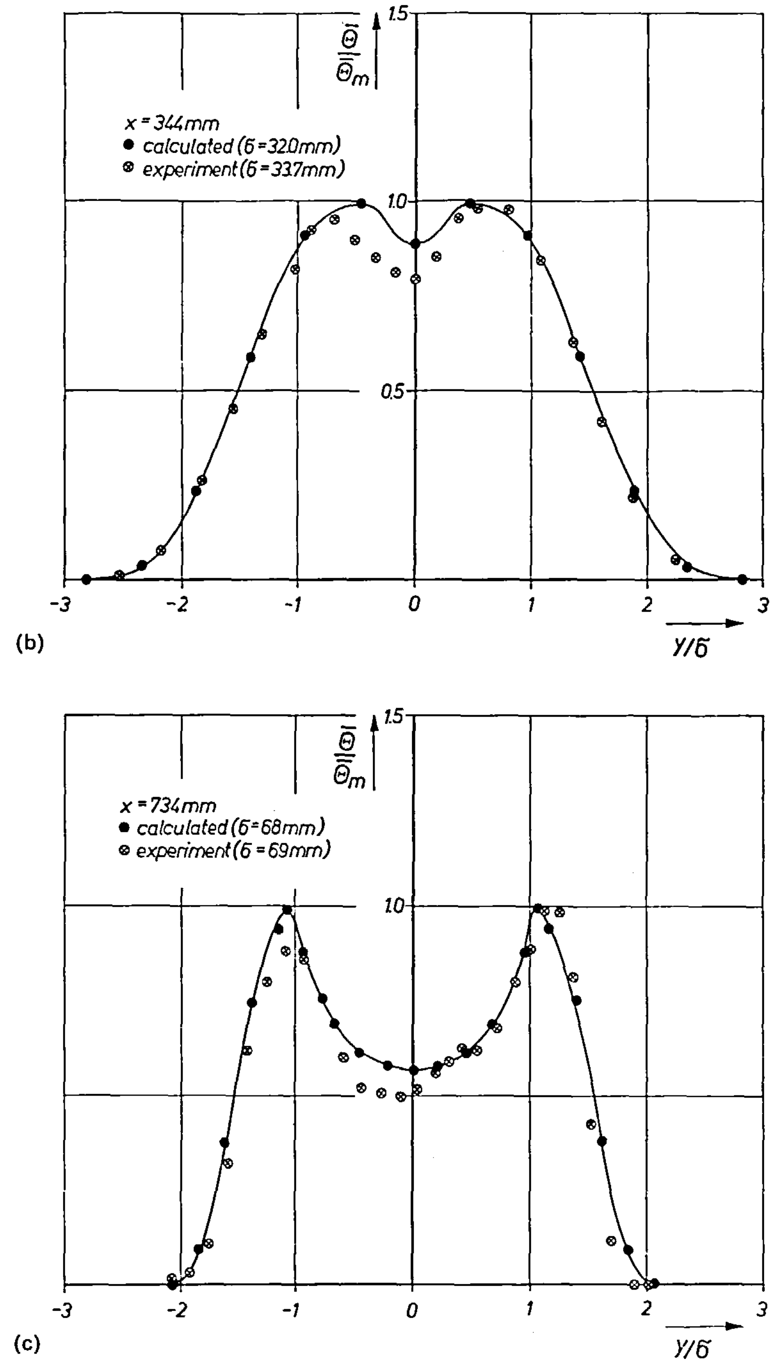

FIG. 7(a)–(c). Comparison of calculated and measured mean temperature profiles, $S = 0.126$.

proportional to the x-coordinate if domains are excluded where the temperature profile is influenced by the arrangement of the wires of the heat source. Figure 8 shows that these domains behind the heat source increase with a decreasing rate of heat spread. Thus the flow with the small Strouhal number $S = 0.0648$ has not reached the region with $\sigma \sim x$ inside the test section of the used wind tunnel. Usual turbulent flows show a heat spread with $\sigma \sim x$, which is advantageous for an effective transfer of heat or matter, only at very short times with $t_D = x/\bar{u}_\infty$ small compared with the Lagrangian integral time scale of the velocity fluctuations v'. From this point of view, the oscillating flow may be regarded as a turbulent one with large integral scales in the y-direction.

Figure 8 demonstrates also that at short diffusion times, the parallel cylinder grid has a high capability to spread heat. This is caused (a) by the high turbulent intensities near the cylinder grid and (b) because, according to Taylor's theory, the rate of heat spread is proportional to the distance x for very short diffusion times in turbulence generated by stationary grids. The

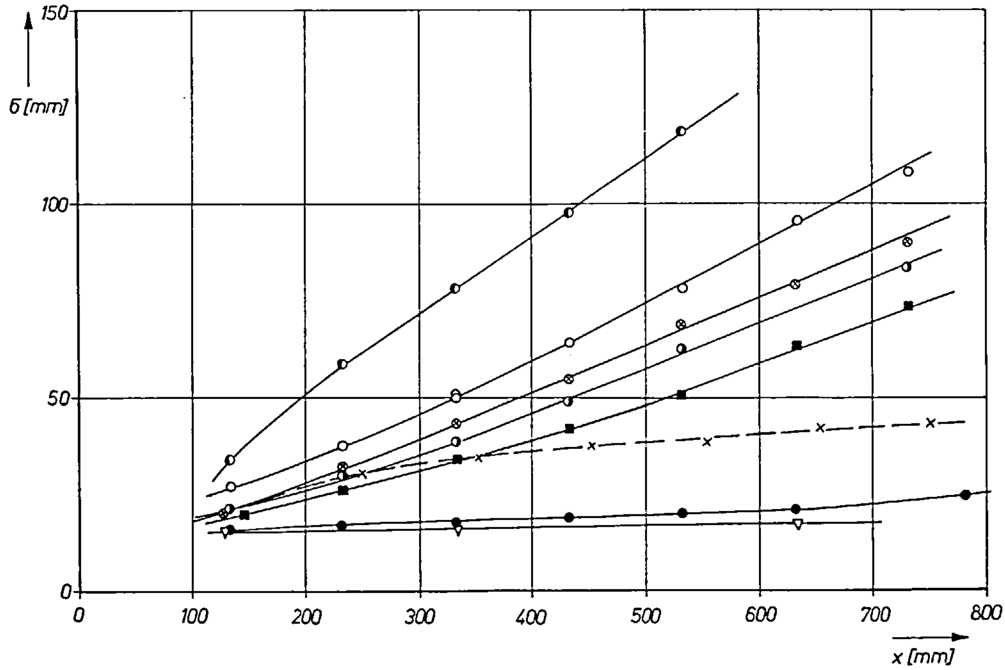

FIG. 8. The standard deviation as a function of the coordinate x. (The symbols are explained in Table 1.)

combination of these two effects provides the high standard deviations near the heat source.

The question arises next of how to determine the Strouhal number at which the mixing behind the oscillating grid is most efficient. Let us consider domains of the oscillating flows with σ proportional to x. As the Lagrangian integral time scale of the velocity fluctuations, v', is large, we can assume that the decay of these velocity fluctuations along the x-axis is small. Thus roughly homogeneous turbulence can be supposed. If we take into account that

$$t_D = \frac{x}{\bar{u}_\infty} \ll \int_0^\infty \frac{\langle v'(0)v'(t)\rangle}{\langle v'^2\rangle}\,dt,$$

the standard deviation $\langle y^2\rangle^{1/2}$ of the particles from their initial position ($t_D = 0$) is porportional to t_D so that

$$\langle y^2\rangle^{1/2} = \langle v'^2\rangle^{1/2} t_D. \qquad (9)$$

If the molecular diffusion is neglected and we assume that the used heat source may be regarded as the initial point then

$$\langle y^2\rangle = \sigma^2.$$

An ideal oscillating grid would not produce additional turbulent velocity fluctuations. Moreover, the velocity v of the flow at the trailing edge of the vanes would be equal to the instantaneous velocity v_G of the grid. Therefore

$$v' = v_G = \hat{v}_G \sin \omega t \qquad (10)$$

Table 1. Explanation of the symbols used in Figs. 8 and 9

Symbol	Strouhal number	$\bar{u}_\infty$ [m/s]	f [Hz]	r [mm]
■	0.126	4.48	10	9
○	0.206	4.57	16.67	9
●	0.0648	4.36	5	9
◐	0.374	2.52	16.67	9
◑	0.156	4.35	16.67	6.5
⊗	0.197	2.59	12.5	6.5
▼	0	4.4	0	0
×	-	8.5	parallel cylinder grid [5]	

and with

$$\langle y^2 \rangle = \sigma^2$$

equation (9) becomes

$$\sigma = \hat{v}_G \langle \sin^2 \omega t \rangle^{1/2} t_D. \quad (11)$$

As

$$S = \frac{\hat{v}_G}{\bar{u}_\infty}$$

then

$$\frac{1}{\bar{u}_\infty} \frac{\partial \sigma}{\partial t_D} = \frac{S}{\sqrt{2}}. \quad (12)$$

Of course, equation (12) has to be used with caution in the investigated oscillating flows because the dimensions of the heat source in the x, y plane are not negligible against the x- and y-coordinates of the points considered in the flow field. It seems possible, however, to compare, by means of equation (12), the deviations of the oscillating flows for different S values from the heat spread behind the ideal grid introduced. The same argument can be used if molecular diffusion is considered. However, the influence of molecular diffusion is probably small in the flows investigated. Figure 9 shows the values

$$\frac{\partial \sigma}{\partial x} = \frac{1}{\bar{u}_\infty} \frac{\partial \sigma}{\partial t_D}$$

which were calculated from the curves in Fig. 8 in the domain with $\sigma \sim x$ as a function of S. The dashed line corresponds to the heat spread behind the ideal oscillating grid which is described by equation (12). Deviations from the ideal case occur at high S. The grid used operates most effectively at Strouhal numbers smaller than about 0.22. The quantitative correspondence to equation (12) must be treated with caution because of the simplifications which have been used. The relatively small rate of heat spread at large S is probably caused by flow separation at the grid vanes when they are strongly inclined to the mean velocity.

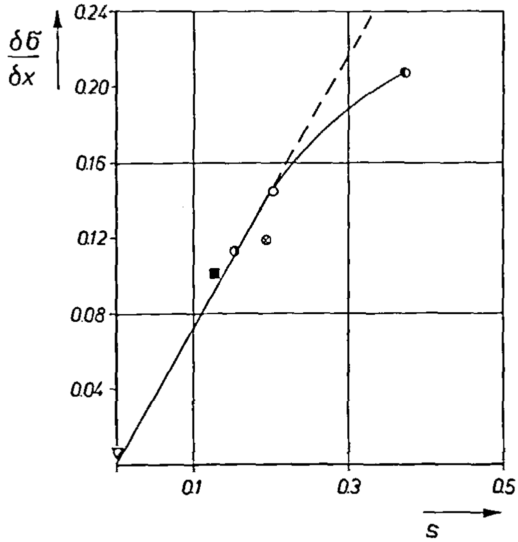

FIG. 9. Comparison of the rate of heat spread in the investigated oscillating flows (---- 'ideal' grid).

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ETUDE DU TRANSFERT THERMIQUE DERRIERE UNE GRILLE OSCILLANTE

Résumé—Des mesures de transfert thermique derrière une grille qui oscille harmoniquement donnent les flux de chaleur qui diffusent lesquels sont plus importants que dans le cas usuel des écoulements turbulents à travers les grilles stationnaires. Le profils de température moyenne montrent une forme inattendue avec deux maximums et un minimum entre eux. Ceci est causé par des fluctuations de vitesse de large échelle qui produisent des flux thermiques indépendants du gradient local de température moyenne. On propose un modèle simple qui prend en compte à la fois les fluctuations de vitesse de grande échelle produites par la grille oscillante et la turbulence 'usuelle'. Les profils calculés de température moyenne montrent un accord convenable avec les mesures.

UNTERSUCHUNGEN ZUM WÄRMETRANSPORT HINTER EINEM SCHWINGENDEN GITTER

Zusammenfassung—Messungen des Wärmetransportes hinter einem harmonisch schwingenden Gitter ergaben eine schnellere Wärmeausbreitung als in herkömmlichen turbulenten Strömungen hinter stationären Gittern. Die Verteilungen der mittleren Temperatur wiesen eine unerwartete Form mit zwei Maxima und einem dazwischen befindlichen Minimum auf. Als Ursache sind Geschwindigkeitsfluktuationen großer räumlicher Ausdehnung anzusehen, die vom lokalen Gradienten der mittleren Temperatur unabhängige Wärmeströme hervorrufen. Es wird ein einfaches Wärmetransportmodell vorgeschlagen, welches sowohl die vom Schwingitter erzeugten Geschwindigkeitsfluktuationen großer räumlicher Ausdehnung als auch die 'herkömmliche' Turbulenz berücksichtigt. Zwischen den berechneten und den gemessenen Verteilungen der mittleren Temperatur besteht gute Übereinstimmung.

ИССЛЕДОВАНИЕ ТЕПЛОПЕРЕНОСА ЗА ВИБРИРУЮЩЕЙ СЕТКОЙ

Аннотация—Измерения теплопереноса за сеткой, совершающей гармонические колебания, позволили определить скорости распространения тепла, значения которых оказались выше значений, полученных в обычных потоках за неподвижными сетками. Средние профили температуры имеют необычную форму с двумя максимумами, разделенными минимумом, что связано с наличием крупномасштабных колебаний скорости, вызывающих тепловые потоки, не зависящие от локального градиента осредненной температуры. Предложена простая модель теплопереноса, в которой учитываются как крупномасштабные регулярные пульсации скорости, возникающие в результате вибрации сетки, так и 'обычная' турбулентность. Рассчитанные средние профили температур хорошо согласуются с измеренными значениями.